Evaluating Exponential and Logarithmic Functions

Use the definition of a logarithm to write the given equation in logarithmic form.

1. $5^3 = 125$ $\log_5 125 = 3$
2. $6^{-2} = \frac{1}{36}$ $\log_6 \frac{1}{36} = -2$
3. $e^3 = 20.085$ $\ln 20.085 = 3$
4. $e^x = 4$ $\ln 4 = x$
5. $8^2 = 64$ $\log_8 64 = 2$
6. $81^{1/4} = 3$ $\log_{81} 3 = \frac{1}{4}$
7. $10^{-3} = 0.001$ $\log 0.001 = -3$
8. $e^0 = 1$ $\ln 1 = 0$
9. $u^v = w$ $\log_u w = v$
10. $9^{3/2} = 27$ $\log_9 27 = \frac{3}{2}$

Use the definition of a logarithm to write the given equation in exponential form.

11. $\log_2 8 = x$ $2^x = 8$
12. $\log_5 625 = 4$ $5^4 = 625$
13. $\log_x 13 = 5$ $x^5 = 13$
14. $\log_2 \frac{1}{8} = -3$ $2^{-3} = \frac{1}{8}$
15. $\log_4 64 = 3$ $4^3 = 64$
16. $\ln 143 = x$ $e^x = 143$
17. $\log 1000 = 3$ $10^3 = 1000$
18. $\ln x = 14$ $e^{14} = x$
19. $\log \frac{1}{100} = -2$ $10^{-2} = \frac{1}{100}$
20. $\ln 18 = x$ $e^x = 18$

Use your calculator to evaluate the following. Round to three decimal places.

21. log 68 = 1.833
22. log 100 = 2
23. ln 9 = 2.197
24. log 10 = 1
25. ln 216 = 5.375
26. ln 9548 = 9.164
27. log 0.0001 = -4
28. log 17 = 1.230
29. ln 125 = 4.828
30. log 6158 = 3.789

Use the change of base formula to evaluate. Round to three decimal places.

$\log_3 7 = \dfrac{\ln 7}{\ln 3}$ *or* $\dfrac{\log 7}{\log 3}$

31. $\log_3 7 = 1.771243749$
$\log_3 7 = 1.771$

$\log_9 0.4 = \dfrac{\ln 0.4}{\ln 9}$ *or* $\dfrac{\log 0.4}{\log 9}$

32. $\log_9 0.4 = -0.4170218836$
$\log_9 0.4 = -0.4170$

$\log_7 4 = \dfrac{\ln 4}{\ln 7}$ *or* $\dfrac{\log 4}{\log 7}$

33. $\log_7 4 = 0.7124143742$
$\log_7 4 = 0.712$

$\log_{20} 125 = \dfrac{\ln 125}{\ln 20}$ *or* $\dfrac{\log 125}{\log 20}$

34. $\log_{20} 125 = 1.611730721$
$\log_{20} 125 = 1.612$

$\log_6 95 = \dfrac{\ln 95}{\ln 6}$ *or* $\dfrac{\log 95}{\log 6}$

35. $\log_6 95 = 2.541567085$
$\log_6 95 = 2.542$

$\log_{0.5} 4 = \dfrac{\ln 4}{\ln 0.5}$ *or* $\dfrac{\log 4}{\log 0.5}$

36. $\log_{0.5} 4 = -2$

$\log_{15} 1250 = \dfrac{\ln 1250}{\ln 15}$ *or* $\dfrac{\log 1250}{\log 15}$

37. $\log_{15} 1250 = 2.63322254$
$\log_{15} 1250 = 2.6332$

$\log_4 0.55 = \dfrac{\ln 0.55}{\ln 4}$ *or* $\dfrac{\log 0.55}{\log 4}$

38. $\log_4 0.55 = -0.4312482381$
$\log_4 0.55 = -0.4312$

$\log_{1/3} 0.015 = \dfrac{\ln 0.015}{\ln 1/3}$ *or* $\dfrac{\log 0.015}{\log 1/3}$

39. $\log_{1/3} 0.015 = 3.822736302$
$\log_{1/3} 0.015 = 3.8227$

$\log_{17} 2 = \dfrac{\ln 2}{\ln 17}$ *or* $\dfrac{\log 2}{\log 17}$

40. $\log_{17} 2 = 0.2446505421$
$\log_{17} 2 = 0.2447$

Properties of Logarithms

Use the properties of logarithms to expand the following.

1. $\log_2 5x = \log_2 5 + \log_2 x$

2. $\log_8 x^4 = 4\log_8 x$

3. $\log_3 \frac{5}{x} = \log_3 5 - \log_3 x$

4. $\ln \sqrt{z} = \ln z^{1/2} = \frac{1}{2}\ln z$

5. $\ln \sqrt{z}\,(z-1)^2 = \frac{1}{2}\ln z + 2\ln(z-1)$

6. $\log_7 \frac{x^2}{y^2 z^3} = 2\log_7 x - 2\log_7 y - 3\log_7 z$

7. $\log\left(\frac{x^2-1}{x^3}\right)^3 = 3\log(x+1) + 3\log(x-1) - 9\log x$

8. $\log_x \frac{\sqrt{a}\,y^4}{z^4} = \frac{1}{2}\log_x a + 4\log_x y - 4\log_x z$

9. $\ln \frac{x}{\sqrt{x^2+1}} = \ln x - \frac{1}{2}\ln\left(x^2+1\right)$

10. $\log\left(x^2 - 8x + 15\right) = \log(x-5) + \log(x-3)$

Use the properties of logarithms to rewrite the following as a single logarithm.

11. $\ln x + \ln 2 = \ln 2x$

12. $\log_4 z - \log_4 y = \log_4 \frac{z}{y}$

13. $2\log_2(x+4) = \log_2(x+4)^2$

14. $\frac{1}{3}\log_3 5x = \log_3 \sqrt[3]{5x}$

15. $\log_3(x-2) - \log_3(x+2) = \log_3 \frac{x-2}{x+2}$

16. $2\ln 8 + 5\ln z = \ln 64z^5$

17. $3\ln x + 2\ln y - 4\ln z = \ln \frac{x^3 y^2}{z^4}$

18. $4\left[\ln z + \ln(z+5)\right] - 2\ln(z-5) = \ln \frac{z^4(z+5)^4}{(z-5)^2}$

19. $\ln x - 2\left[\ln(x+2) + \ln(x-2)\right] = \ln \frac{x}{\left(x^2-4\right)^2}$

20. $\frac{3}{2}\log_4 5t^6 - \frac{3}{4}\log_4 t^4 = \log_4 5\sqrt{5}\,t^6$

Solving Exponential and Logarithmic Equations

Solve the following exponential equations.

1. $10^x = 42$

$\log 42 = x$

$x \approx 1.62324929$

$x \approx 1.623$

2. $\frac{1}{3}\left(10^{2x}\right) = 12$

$10^{2x} = 36$

$\log 36 = 2x$

$\frac{\log 36}{2} = x$

$x \approx 0.7781512504$

$x \approx 0.778$

3. $e^x = 10$

$\ln 10 = x$

$x \approx 2.302585093$

$x \approx 2.303$

4. $25e^{2x+1} = 962$

$e^{2x+1} = 38.48$

$\ln 38.48 = 2x + 1$

$(\ln 38.48) - 1 = 2x$

$\frac{(\ln 38.48) - 1}{2} = x$

$x \approx 1.325069313$

$x \approx 1.325$

5. $1000e^{-4x} = 75$

$e^{-4x} = 0.075$

$\ln 0.075 = -4x$

$\frac{\ln 0.075}{-4} = x$

$x \approx 0.6475667914$

$x \approx 0.648$

6. $\frac{1250}{1.04^x} = 500$

$\frac{1250}{1.04^x} = \frac{500}{1}$

$500\left(1.04^x\right) = 1250$

$1.04^x = 2.5$

$\log_{1.04} 2.5 = x$

$x = \frac{\ln 2.5}{\ln 1.04}$ *or* $\frac{\log 2.5}{\log 1.04}$

$x \approx 23.36241894$

$x \approx 23.362$

Solve the following logarithmic equations. (Round answers to three decimal places)

7. $2\ln x = 7$

$\ln x = 3.5$

$e^{3.5} = x$

$x \approx 33.11545196$

$x \approx 33.115$

8. $\log(x-3) = 2$

$10^2 = x - 3$

$100 = x - 3$

$x = 103$

9. $6\ln(x+1)=2$

$\ln(x+1)=\frac{1}{3}$

$e^{1/3}=x+1$

$e^{1/3}-1=x$

$x\approx 0.3956124251$

$x\approx 0.396$

10. $\log 2+\log x=3$

$\log 2x=3$

$10^3=2x$

$1000=2x$

$x=500$

11. $\ln x+\ln(x-2)=1$

$\ln\left[x(x-2)\right]=1$

$e^1=x(x-2)$

$e=x^2-2x$

Solve by completing the square.

$e+1=x^2-2x+1$

$e+1=(x-1)^2$

$\pm\sqrt{e+1}=x-1$

$1\pm\sqrt{e+1}=x$

$x=1+\sqrt{e+1}$ *or* $x=1-\sqrt{e+1}$

$x\approx 2.928284686$ *or* $x\approx -0.9282846855$

$x\approx 2.928$ *or* $x\approx -0.928$

Since you can't do the ln *of a negative number,*

$x\approx 2.928$

12. $\log_4 x - \log_4 (x-1) = \frac{1}{2}$

$\log_4 \frac{x}{x-1} = \frac{1}{2}$

$4^{1/2} = \frac{x}{x-1}$

$\sqrt{4} = \frac{x}{x-1}$

$\frac{2}{1} = \frac{x}{x-1}$

$2(x-1) = x$

$2x - 2 = x$

$x - 2 = 0$

$x = 2$