Rational Equations and Functions
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(17) Chapter Review ***Test Tomorrow***

Supp w/ Algebra 1 (Ch 11) - determine how a change in one variable relates to a change in a second

Two variables x and y show direct variation provided $y=k x$ and $k \neq 0$. The nonzero constant k is called the constant of variation and y is said to vary directly with x . The graph of $y=k x$ is a line through the origin.

E1. The variables $x$ and $y$ vary directly, and $y=12$, when $x=4$.
a. Write an equation relating $x$ and $y$
b. Find y when $\mathrm{x}=5$.

P1. The variables $x$ and $y$ vary directly, and $y=15$, when $x=3$
a. Write an equation relating $x$ and $y$
b. Find y when $\mathrm{x}=3$.

E2. Tell whether the given data shows direct variation. If so, write an equation relating $x$ and $y$

a. | Time,$x$ (hours) | 3 | 6 | 9 | 12 | 15 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Wages, $y$ (dollars) | 12 | 24 | 38 | 50 | 65 |

b • | Weight, $x$ (ounces) | 8 | 16 | 24 | 32 | 40 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price of nuts, $y$ (dollars) | 2.40 | 4.80 | 7.20 | 9.60 | 12.00 |

P2. Tell whether the given data shows direct variation. If so, write an equation relating $x$ and $y$
a. 14-karat Gold Chains (1 gram per inch)

| Length, x <br> (inches) | 16 | 18 | 20 | 24 | 30 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price, y <br> (dollars) | 288 | 324 | 360 | 432 | 540 |

b. Loose Diamonds (round, colorless, very small flaws)

| Weight, x <br> (carats) | 0.5 | 0.7 | 1.0 | 1.5 | 2.0 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Price, y <br> (dollars) | 2250 | 3430 | 6400 | 11,000 | 20,400 |

Another type of variation is called inverse variation. Two variables $x$ and $y$ show inverse variation if they are related as follows: $y=\frac{k}{x}, k \neq 0$

The nonzero constant $k$ is called the constant of variation, and $y$ is said to vary inversely with $x$.
Classifying direct and inverse variation
E1. Tell whether $x$ and $y$ show direct variation, inverse variation or neither

| Given Equation | Rewritten Equation | Type of Variation |
| :---: | :---: | :---: |
| a. $\frac{y}{5}=x$ | $\mathrm{y}=5 \mathrm{x}$ | Direct |
| b. $\mathrm{y}=\mathrm{x}+2$ |  | Neither |
| c. $\mathrm{xy}=4$ | $y=\frac{4}{x}$ | Inverse |
|  |  |  |

P1. Tell whether $x$ and $y$ show direct variation, inverse variation or neither

| Given Equation | Rewritten Equation | Type of Variation |
| :---: | :---: | :---: |
| a. $x y=12$ |  |  |
| b. $\frac{y}{10}=x$ |  |  |
| c. $x+y=20$ |  |  |

Writing an Inverse Variation Equation

E2. The variables $x$ and $y$ vary inversely, and $y=8$ when $x=3$.
a. Write an equation that relates $x$ and $y$
b. Find $y$ when $x=-4$

P2. The variables $x$ and $y$ vary inversely, and $y=6$ when $x=1.5$
a. Write an equation that relates $x$ and $y$
b. Find y when $\mathrm{x}=\frac{4}{3}$

E3. The speed of the current in a whirlpool varies inversely with the distance from the whirlpool's center. The Lofoten Maelstrom is a whirlpool located off the coast of Norway. At a distance of 3 kilometers ( 3000 meters) for the center, the speed of the current is about 0.1 meter per second. Describe the change in the speed of the current as you move closer to the whirlpool.

The equation for inverse variation can be rewritten as $x y=k$. This tells you that a set of data points ( $x, y$ ) shows inverse variation if the products $x y$ are constant or approximately constant.

P3. The ideal gas law states that the volume V (in liters) varies directly with the number of molecules $n$ (in moles) and temperature $T$ (in Kelvin) and varies inversely with the pressure $P$ (in kilopascals). The constant of variation is denoted by $R$ and is called the universal gas constant.
a. Write an equation for the ideal gas law.
b. Estimate the universal gas constant if $\mathrm{V}=251.6$ liters; $\mathrm{n}=1$ mole; $\mathrm{T}=288 \mathrm{~K} ; \mathrm{P}=9.5$ kilopascals.

## Simplest form

- a rational expression is in simplest form provided it's numerator and denominator have no common factors (other than 1 or -1 )


## Reminder

- Don't forget that division is multiplication of the reciprocal. Must re-write any division as multiplication before reducing.


## Steps:

1) Factor all numerators and denominators
2) Reduce any common factors
a. Top to bottom (Reducing)
b. Diagonally (Cross Reducing)
3) Multiply

E1. $\frac{x^{2}-4 x-12}{x^{2}-4}$
P1. $\frac{x^{2}-5 x-6}{x^{2}-1}$

E2. $\frac{5 x^{2} y}{2 x y^{3}} \bullet \frac{6 x^{3} y^{2}}{10 y}$
P2. $\quad \frac{6 x^{2} y^{3}}{2 x^{2} y^{2}} \bullet \frac{10 x^{3} y^{4}}{18 y^{2}}$

E3. $\frac{4 x-4 x^{2}}{x^{2}+2 x-3} \bullet \frac{x^{2}+x-6}{4 x}$
P3. $\frac{3 x-27 x^{3}}{3 x^{2}-2 x-1} \cdot \frac{3 x^{2}-4 x+1}{3 x}$

E4. $\frac{5 x}{3 x-12} \div \frac{x^{2}-2 x}{x^{2}-6 x+8}$

E5. $\quad \frac{6 x^{2}+7 x-3}{6 x^{2}} \div\left(2 x^{2}+3 x\right)$

E6. $\frac{x}{x+5} \bullet(3 x-5) \div \frac{9 x^{2}-25}{x+5}$

P4. $\frac{3}{4 x-8} \div \frac{x^{2}+3 x}{x^{2}+x-6}$

P5. $\frac{8 x^{2}+10 x-3}{4 x^{2}} \div\left(4 x^{2}-x\right)$

Steps:
(1) CHANGE - All subtraction to addition (subtraction is simply addition of the opposite)
(2) FACTOR - All Numerators and Denominators
(3) MAKE COMMON DENOMINATORS - Multiply the numerators out
(4) COMBINE - Combine the numerators like-terms in descending order over one common denominator.
(5) REDUCE - Re-factor the new numerator and reduce with common factors in the denominator.

Directions: Simplify each rational expression completely.

E1. $\frac{4}{3 x}+\frac{5}{3 x}$

E2. $\frac{2 x}{x+3}+\frac{-4}{x+3}$

E3 $\cdot \frac{5}{6 x^{2}}+\frac{x}{4 x^{2}-12 x}$
P3. $\frac{4}{3 x^{3}}+\frac{x}{6 x^{3}+3 x^{2}}$

E4. $\frac{x+1}{x^{2}+4 x+4}-\frac{2}{x^{2}-4}$

E5. $\frac{5}{x-6}-\frac{20}{x^{2}-4 x-12}$

E6. $\frac{\frac{2}{x+2}}{\frac{1}{x+2}+\frac{2}{x}}$

P4. $\frac{x+1}{x^{2}+6 x+9}-\frac{1}{x^{2}-9}$

P5. $\frac{6}{x^{2}-4 x-5}+\frac{1}{x+1}$

P6. $\frac{\frac{3}{x-4}}{\frac{1}{x-4}+\frac{3}{x+1}}$

Steps:
(1) FACTOR - All denominators
(2) CLEAR FRACTIONS - Multiply both sides by the LCD
(3) SOLVE - The resulting equation

* Cross Multiply whenever possible*
(4) CHECK - Check the solution and watch for division by zero

Directions: Solve the rational equation. Check your solution.
E1. $\frac{4}{x}+\frac{5}{2}=\frac{11}{x}$
P1. $\frac{3}{x}-\frac{1}{2}=\frac{12}{x}$

E2. $\frac{5 x}{x-2}=7+\frac{10}{x-2}$
P2. $\frac{5 x}{x+1}=4-\frac{5}{x+1}$

$$
\text { E3 } \quad \frac{4 x+1}{x+1}=\frac{12}{x^{2}-1}+3
$$

P3. $\frac{3 x-2}{x-2}=\frac{6}{x^{2}-4}+1$

E4. $\frac{2}{x^{2}-x}=\frac{1}{x-1}$
P4. $\frac{3}{x^{2}+4 x}=\frac{1}{x+4}$

## (1) WORK PROBLEMS <br> a. Set up a chart (work rate $\bullet$ time = work done) <br> b. Write an equation <br> i. Working together (work done + work done =1) <br> ii. Working apart (work done - work done = 1) <br> c. Solve the equation <br> (2) WORDY WORD PROBLEMS

Directions: Solve the rational equation. Check your solution.

## E1. Work Problem:

My wife can wash my car in 2 hours. My brother's wife can wash my car in 3 hours. If they work together, how long will it take them to wash my car?

## P1. Work Problem:

Shawna can pour a large concrete driveway in six hours. Dan can pour the same driveway in seven hours. Find out how long it would take them if they worked together.

## E2. Work Problem:

Tracey and Alan publish a 10-page independent newspaper once a month. At production, Alan usually spends 6 hours on the layout of the paper. When Tracey helps, layouts take 3 hours and 20 minutes. How long would it take Tracy to do the job alone?

## P2. Work Problem:

Working together, Paul and Daniel can pick forty bushels of apples in 4.95 hours. Had he done it alone it would have taken Daniel 9 hours. Find out how long it would take Paul to do it alone?

## E3. Wordy Word Problem:

One half a number increased by seven is two-fifths of the same number. Find the number.

## P3. Wordy Word Problem:

Three fourths of a number increased by eight is forty one. Find the number.

## E4. Distance- Rate- Time Problem:

An aircraft carrier made a trip to Guam and back. The trip there took three hours and the trip back took four hours. It averaged $6 \mathrm{~km} / \mathrm{h}$ on the return trip. Find the average speed of the trip there.

## P4. Distance- Rate- Time Problem:

A passenger place made a trip to Las Vegas and back. On the trip there it flew 432 mph and on the return trip it went 480 mph . How long did the trip take it the return trip took nine hours?

